Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

In particular, if the circles having UU' and WW' as diameters intersect at the points P and Q,

$$BP : PC = BA : AC \text{ and } BQ : QC = BA : AC.$$

Similarly,

$$AP : PB = AC : CB \text{ and } AQ : QB = AC : CB.$$

Hence,

$$AP : PC = AB : BC \text{ and } AQ : QC = AB : BC.$$

It follows that P and Q are points on the circle having VV' as diameter. Hence the three circles UU', VV', WW' are coaxial.

2. Since (BC, UU') is a harmonic range, any circle passing through B and C is orthogonal to the circle having UU' as diameter. Hence, the circumcircle is orthogonal to that circle. Similarly the circumcircle is orthogonal to the circles VV' and WW'.

Hence, if O be the circumcenter, OA is a tangent to UU', OB to VV' and OC to WW'. But these tangents are equal in length.

Hence O is a point on the common chord of the circles UU', VV', WW'.

Also solved by C. E. Horne, David F. Kelley, Elijah Swift, Roger A. Johnson, and C. N. Schmall.

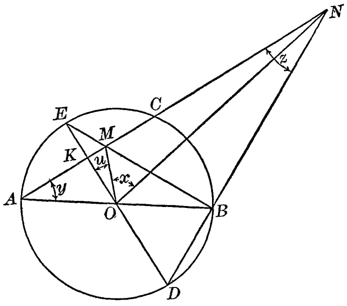
457. Proposed by NATHAN ALTSHILLER, University of Washington.

AB and AC are respectively a diameter and a chord of a circle whose center is O. The lines joining B to the extremities of the diameter perpendicular to AC, meet AC in the points M, N. Express the angle MON in terms of the angle CAB.

Solution by J. A. Caparo, University of Notre Dame.

Let E and D be the extremities of the diameter perpendicular to AC at the point K. With the notation of the figure we easily see that

$$\sin \angle AMB = \sin \angle ABN = \cos z.$$



Hence, by the sine law, letting $AB = 2R$, we have in $\triangle ANB$, $AN = 2R \cot z$, and in $\triangle AMB$, $AM = 2R \tan z$. Also from $\triangle AKO$, $AK = R \cos y$, $OK = R \sin y$. $KN = AN - AK = 2R \cot z - R \cos y$, $MK = AM - AK = 2R \tan z - R \cos y$.

$$\therefore \tan u = \frac{MK}{OK} \frac{2 \tan z - \cos y}{\sin y}. \quad \text{Also } \tan (x + u) = \frac{KN}{KO}.$$

Hence, substituting the values of KN and OK,

$$\frac{\tan x + \tan u}{1 - \tan x \tan u} = \frac{2 \cot z - \cos y}{\sin y}.$$

Substituting the value of $\tan u$,

$$2 \sin y(\tan z - \cot z) = 2 \tan x \cos y(\cot z + \tan z) - 5 \tan x.$$

But $\tan z - \cot z = -2 \tan y$ and $\cot z + \tan z = \dfrac{2}{\cos y}$.

Hence, substituting and reducing,

$$\tan x = 4 \tan y \sin y,$$

the required relation.

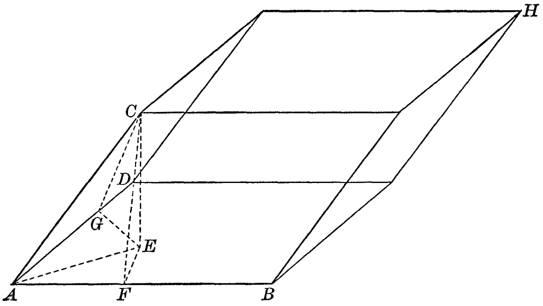
Also solved by Paul Capron and J. W. Clawson.

458. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Given edges l, m and n of a parallelopiped and angles a, b and c which the edges make with one another. Show that, if $s = \dfrac{a + b + c}{2}$, the volume equals

$$2lmn\sqrt{\sin s \sin (s - a) \sin (s - b) \sin (s - c)}.$$

Solution by Frank R. Morris, Glendale, Cal.



Given the parallelopiped AH with $\angle BAC = a$, $\angle DAC = b$, $\angle BAD = c$, and $AB = l$, $AC = m$, $AD = n$.

The area of the base BD is equal to $ln \sin C$.

From the vertex C drop a perpendicular to the base BD meeting it at E. From E draw perpendiculars to AB and AD meeting the lines at F and G, respectively. CE is the altitude of the parallelopiped and we know that the volume is $V = CE \cdot ln \sin c$. (1).

The triangles AEC, AFC, AGC, AFE and AGE are right triangles. Hence, we have

$$CE^2 = m^2 - AE^2, \quad (2) \qquad AF = m \cos a, \; AG = m \cos b, \quad (3)$$

$$\cos \angle EAF = \frac{AF}{AE}, \quad (4) \qquad \text{and} \qquad \cos (c - \angle EAF) = \frac{AG}{AE},$$

or

$$\cos c \cos \angle EAF + \sin c \sqrt{1 - \cos^2 \angle EAF} = \frac{AG}{AE}. \qquad (5)$$

Eliminating $\cos \angle EAF$ from (4) and (5)

$$\cos c \frac{AF}{AE} + \sin c \sqrt{1 - \frac{AF^2}{AE^2}} = \frac{AG}{AE}.$$